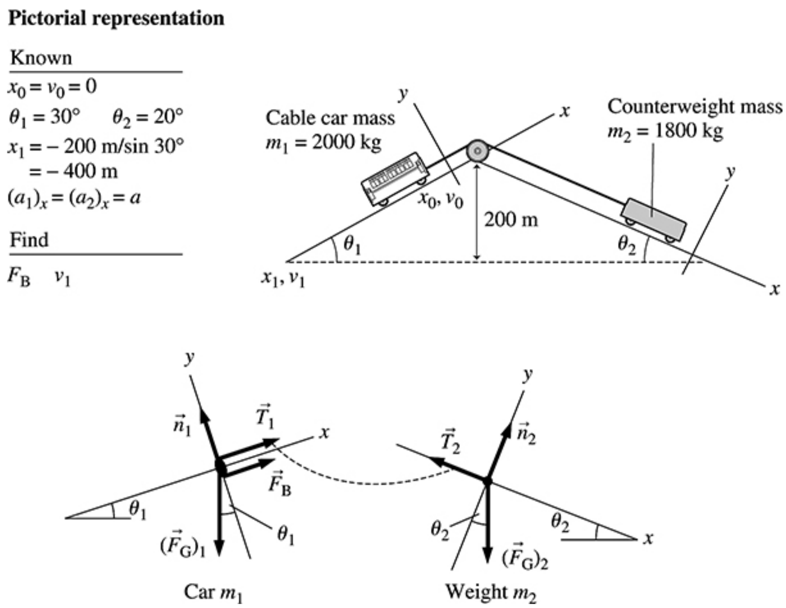
7.42. Model: Use the particle model for the cable car and the counterweight. Assume a massless cable.
Visualize:



Solve: **(a)** Notice the separate coordinate systems for the cable car (object 1) and the counterweight (object 2). Forces $\vec{T}_1$ and $\vec{T}_2$ act as if they are an action/reaction pair. The braking force $\vec{F}_B$ works with the cable tension $\vec{T}_1$ to allow the cable car to descend at a constant speed. Constant speed means dynamic equilibrium, so $\vec{F}_{net} = 0$ N for both systems. Newton's second law for the cable car is

$$\left(F_{\text{net on 1}}\right)_x = T_1 + F_B - m_1 g \sin\theta_1 = 0 \text{ N} \quad \left(F_{\text{net on 1}}\right)_y = n_1 - m_1 g \cos\theta_1 = 0 \text{ N}$$

Newton's second law for the counterweight is

$$\left(F_{\text{net on 2}}\right)_x = m_2 g \sin\theta_2 - T_2 = 0 \text{ N} \quad \left(F_{\text{net on 2}}\right)_y = n_2 - m_2 g \cos\theta_2 = 0 \text{ N}$$

From the x-equation for the counterweight, $T_2 = m_2 g \sin\theta_2$. By Newton's third law, $T_1 = T_2$. Thus the x-equation for the cable car becomes

$$F_B = m_1 g \sin\theta_1 - T_1 = m_1 g \sin\theta_1 - m_2 g \sin\theta_2 = 3770 \text{ N}$$

(b) If the brakes fail, then $F_B = 0$ N. The car will accelerate down the hill on one side while the counterweight accelerates up the hill on the other side. Both will have *negative* accelerations because of the direction of the acceleration vectors. The constraint is $a_{1x} = a_{2x} = a$, where a will have a negative value. Using $T_1 = T_2 = T$, the two x-equations are

$$\left(F_{\text{net on 1}}\right)_x = T - m_1 g \sin\theta_1 = m_1 a_{1x} = m_1 a \quad \left(F_{\text{net on 2}}\right)_x = m_2 g \sin\theta_2 - T = m_2 a_{2x} = m_2 a$$

Note that the y-equations aren't needed in this problem. Add the two equations to eliminate T:

$$-m_1 g \sin\theta_1 + m_2 g \sin\theta = \left(m_1 + m_2\right) a \Rightarrow a = -\frac{m_1 \sin\theta_1 - m_2 \sin\theta_2}{m_1 + m_2} g = -0.991 \text{ m/s}^2$$

Now we have a problem in kinematics. The speed at the bottom is calculated as follows:

$$v_1^2 = v_0^2 + 2a\left(x_1 - x_0\right) = 2ax_1 \Rightarrow v_1 = \sqrt{2ax_1} = \sqrt{2\left(-0.991 \text{ m/s}^2\right)\left(-400 \text{ m}\right)} = 28.2 \text{ m/s}$$

Assess: A speed of approximately 60 mph as the cable car travels a distance of 2000 m along a frictionless slope of 30° is reasonable.